SAMPLE QUESTION PAPER

Issued by CBSE for 2018 Examinations

Class XII - Mathematics

Time Allowed : 180 Minutes **Max. Marks : 100**

General Instructions :

(a) All questions are compulsory.

(b) This question paper consist of **29 questions** divided into **four sections A, B, C and D**. Section A comprises of **04 questions of one mark** each, section B comprises of **08 questions of two marks** each, section C comprises of **11 questions of four marks** each and section D comprises of **06 questions of six marks** each.

(c) All the questions in **Section A** are to be answered in one word, one sentence or as per **the exact requirement** of the question.

(d) There is no overall choice. However, **internal choice** has been provided in **03 questions of four marks each** and **03 questions of six marks each**. You have to attempt only one of the alternatives in all such questions.

SECTION A

Q01. Let A = {1, 2, 3, 4}. Let R be the equivalence relation on $A \times A$ defined by (a, b) R (c, d) iff $a + d = b + c$. Find the equivalence class [(1, 3)].

Q02. If A = $[a_{ij}]$ is a matrix of order 2×2, such that |A| = –15 and C_{ij} represents the cofactor of a_{ij}, then find $a_{21}C_{21} + a_{22}C_{22}$.

Q03. Give an example of vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = |\vec{b}|$ but $\vec{a} \neq \vec{b}$.

Q04. Determine whether the binary operation * on the set N of natural numbers defined by a*b = 2^{ab} is associative or not.

SECTION B

Q05. If $4\sin^{-1} x + \cos^{-1} x = \pi$, then find the value of x.

Q06. Find the inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$. Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

Q07. Prove that if $\frac{1}{2} \le x \le 1$ then, $\cos^{-1} x + \cos^{-1}\left[\frac{x}{2} + \frac{\sqrt{3-3x^2}}{2}\right] = \frac{\pi}{3}$.

Q08. Find the approximate change in the value of $\frac{1}{x^2}$, when x changes from $x = 2$ to $x = 2.002$.

Q09. Find $\int e^x \frac{\sqrt{1+\sin 2x}}{1+\cos 2x} dx$.

Q10. Verify that $ax^2 + by^2 = 1$ is a solution of the differential equation $x(yy_2 + y_1^2) = yy_1$.

Q11. Find the projection (vector) of $2\hat{i} - \hat{j} + \hat{k}$ on $\hat{i} - 2\hat{j} + \hat{k}$.

Q12. If A and B are two events such that P(A) = 0.4, P(B) = 0.8 and P(B|A) = 0.6, then find P(A|B).

SECTION C

Q13. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & 1 & a \end{vmatrix} = -4$ then find the value of $\begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}$.

Q14. Find 'a' and 'b', if the function given by $f(x)=\begin{cases} ax^2+b, \text{ if } x<1 \\ 2x+1, \text{ if } x\ge 1\end{cases}$ is differentiable at $x=1$.

OR Determine the values of 'a' and 'b' s.t. the following function is continuous at $x=0$:

$$f(x)=\begin{cases} \dfrac{x+\sin x}{\sin(a+1)x}, \text{ if } -\pi<x<0 \\ 2, \text{ if } x=0 \\ 2\dfrac{e^{\sin bx}-1}{bx}, \text{ if } x>0 \end{cases}.$$

Q15. If $y=\log\left(\sqrt{x}+\dfrac{1}{\sqrt{x}}\right)^2$, then prove that $x(x+1)^2y_2+(x+1)^2y_1=2$.

Q16. Find the equation (s) of the tangent (s) to the curve $y=(x^3-1)(x-2)$ at the points where the curve intersects the x-axis.

OR Find the intervals in which the function $f(x)=-3\log(1+x)+4\log(2+x)-\dfrac{4}{2+x}$ is strictly increasing or strictly decreasing.

Q17. A person wants to plant some trees in his community park. The local nursery has to perform this task. It charges the cost of planting trees by the following formula :
$C(x)=x^3-45x^2+600x$, where x is the number of trees and C(x) is the cost of planting x trees in rupees. The local authority has imposed a restriction that it can plant 10 to 20 trees in one community park for a fair distribution. For how many trees should the person place the order so that he has to spend the least amount? How much is the least amount? **Use calculus to answer these questions. Which value is being exhibited by the person?**

Q18. Find $\int \dfrac{\sec x}{1+\operatorname{cosec} x}dx$.

Q19. Find the particular solution of the differential equation : $ye^y dx=(y^3+2xe^y)dy,\ y(0)=1$.

OR Show that $(x-y)dy=(x+2y)dx$ is a homogeneous differential equation. Also find the general solution of the given differential equation.

Q20. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$, and hence show that $[\vec{a}\ \vec{b}\ \vec{c}]=0$.

Q21. Find the equation of the line which intersects the lines $\dfrac{x+2}{1}=\dfrac{y-3}{2}=\dfrac{z+1}{4}$ and $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ and passes through the point (1, 1, 1).

Q22. Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they came from Bag III?

Q23. Four bad oranges are accidently mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean and variance of the distribution.

SECTION D

Q24. If the function $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=2x-3$ and $g:\mathbb{R}\to\mathbb{R}$ by $g(x)=x^3+5$, then find fog and show that fog is invertible. Also find $(fog)^{-1}$, hence find $(fog)^{-1}$ (9).

OR A binary operation * is defined on the set $\mathbb{R}$ of real numbers by

$a*b=\begin{cases} a, \text{ if } b=0 \\ |a|+b, \text{ if } b\ne 0\end{cases}$. If at least one of a and b is 0, then prove that a * b = b * a.

Check whether * is commutative. Find the identity element for *, if it exists.

Q25. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations :

$3x + 4y + 7z = 14,\ 2x - y + 3z = 4,\ x + 2y - 3z = 0$.

OR If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$, find the inverse of A using elementary row transformations and hence solve the matrix equation : $XA = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$.

Q26. Using integration, find the area in the first quadrant bounded by the curve $y = x|x|$, the circle $x^2 + y^2 = 2$ and the y-axis.

Q27. Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x}\, dx$.

OR Evaluate $\int_{-2}^{2} (3x^2 - 2x + 4)\, dx$ as the limit of a sum.

Q28. Find the distance of point $-2\hat{i} + 3\hat{j} - 4\hat{k}$ from the line $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 3\hat{j} - 9\hat{k})$ measured parallel to the plane $x - y + 2z - 3 = 0$.

Q29. A company produces two different products. One of them needs 1/4 of an hour of assembly work per unit, 1/8 of an hour in quality control work and ₹ 1.2 in raw materials. The other product requires 1/3 of an hour of assembly work per unit, 1/3 of an hour in quality control work and ₹ 0.9 in raw materials. Given the current availability of staff in the company, each day there is at most a total of 90 hours available for assembly and 80 hours for quality control. The first product described has a market value (sale price) of ₹ 9 per unit and the second product described has a market value (sale price) of ₹ 8 per unit. In addition, the maximum amount of daily sales for the first product is estimated to be 200 units, without there being a maximum limit of daily sales for the second product. Formulate and solve graphically the LPP and find the maximum profit.

SOLUTIONS Of SAMPLE PAPER

Mathematics XII (2017-18)

SECTION A

Q01. Let (1, 3) R (x, y) for all $(x, y) \in A \times A$.
That implies, $1 + y = 3 + x$ i.e., $y - x = 2$.
So (x, y) may be (1, 3), (2, 4).
Hence $[(1,3)] = \{(1,3), (2,4)\}$.

Q02. As $A = [a_{ij}]_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

Consider $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{21}C_{21} + a_{22}C_{22}$ (if we expand along R_2

$\therefore |A| = a_{21}C_{21} + a_{22}C_{22} = -15$.

Q03. Let $\vec{a} = \hat{i}$ and $\vec{b} = \hat{j}$. Note that $|\vec{a}| = 1 = |\vec{b}|$ but $\vec{a} \neq \vec{b}$.
Other correct examples should be given full marks.

Q04. $1*(2*3) = 1*2^6 = 1*64 = 2^{64}$, $(1*2)*3 = 2^2*3 = 4*3 = 2^{12}$.
Clearly $1*(2*3) \neq (1*2)*3$. Hence * is not associative.

SECTION B

Q05. $4\sin^{-1}x + \cos^{-1}x = \pi \quad \Rightarrow 3\sin^{-1}x + \sin^{-1}x + \cos^{-1}x = \pi \quad \Rightarrow 3\sin^{-1}x = \pi - \dfrac{\pi}{2}$

$\Rightarrow \sin^{-1}x = \dfrac{\pi}{6} \qquad \Rightarrow x = \sin\dfrac{\pi}{6} \qquad \therefore x = \dfrac{1}{2}$.

Q06. Let $A = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} \quad \Rightarrow |A| = \begin{vmatrix} -3 & 2 \\ 5 & -3 \end{vmatrix} = 9 - 10 = -1$, $adj.A = \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{adj.A}{|A|} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$...(i)

Now $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \quad \Rightarrow P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$ (On post-multiplication with A^{-1}

By (i), we get : $P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} \qquad \therefore P = \begin{bmatrix} 3+10 & 2+6 \\ 6-5 & 4-3 \end{bmatrix} = \begin{bmatrix} 13 & 8 \\ 1 & 1 \end{bmatrix}$.

Q07. Let $\theta = \cos^{-1}x$. Then for all $\in \left[\dfrac{1}{2}, 1\right]$, $\theta \in \left[0, \dfrac{\pi}{3}\right]$. Also $x = \cos\theta$.

Consider LHS : Let $y = \cos^{-1}x + \cos^{-1}\left[\dfrac{x}{2} + \dfrac{\sqrt{3-3x^2}}{2}\right] = \theta + \cos^{-1}\left[\dfrac{\cos\theta}{2} + \dfrac{\sqrt{3}}{2}\sqrt{1-\cos^2\theta}\right]$

$\Rightarrow y = \theta + \cos^{-1}\left[\dfrac{1}{2}\cos\theta + \dfrac{\sqrt{3}}{2}\sin\theta\right] = \theta + \cos^{-1}\left[\cos\left(\theta - \dfrac{\pi}{3}\right)\right]$

$\because 0 \le \theta \le \dfrac{\pi}{3} \qquad \Rightarrow -\dfrac{\pi}{3} \le \theta - \dfrac{\pi}{3} \le 0 \qquad \Rightarrow 0 \le \dfrac{\pi}{3} - \theta \le \dfrac{\pi}{3}$

$\therefore y = \theta + \cos^{-1}\left[\cos\left(\dfrac{\pi}{3} - \theta\right)\right] = \theta + \dfrac{\pi}{3} - \theta = \dfrac{\pi}{3} = RHS$.

Note that $\cos(-A) = \cos A$.

Q08. Let $y=\frac{1}{x^2}$. Then $\frac{dy}{dx}=-\frac{2}{x^3}$.

$\therefore dy=\left(\frac{dy}{dx}\right)_{\text{at } x=2}\times dx=\left(-\frac{2}{2^3}\right)\times 0.002=-\frac{0.002}{4}=-0.0005$.

Hence y is decreased by 0.0005.

\# Note that $x=2$ to $x=2.002$ so, $\Delta x\approx dx=2.002-2=0.002$.

Q09. Let $I=\int e^x\frac{\sqrt{1+\sin 2x}}{1+\cos 2x}dx=\int e^x\frac{\sqrt{(\sin x+\cos x)^2}}{2\cos^2 x}dx=\frac{1}{2}\int e^x\left(\frac{\sin x+\cos x}{\cos^2 x}\right)dx$

$\Rightarrow I=\frac{1}{2}\int e^x\left(\sec x+\sec x\tan x\right)dx=\frac{1}{2}e^x\sec x+C$.

\# Note that we've used $\int e^x\left(f(x)+f'(x)\right)dx=e^x f(x)+c$, here $f(x)=\sec x,\ f'(x)=\sec x\tan x$.

Q10. Here $ax^2+by^2=1$ $\Rightarrow 2ax+2byy_1=0$ $\Rightarrow \frac{yy_1}{x}=-\frac{a}{b}$

$\Rightarrow \frac{x\left(yy_2+y_1y_1\right)-yy_1\times 1}{x^2}=0$ $\Rightarrow x(yy_2+y_1^2)=yy_1$, hence verified.

Q11. As projection vector of $\vec{a}$ on $\vec{b}=(\vec{a}.\hat{b})\hat{b}$

So, projection of $2\hat{i}-\hat{j}+\hat{k}$ on $\hat{i}-2\hat{j}+\hat{k}$ is $\left[(2\hat{i}-\hat{j}+\hat{k}).\left(\frac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{1^2+(-2)^2+1^2}}\right)\right]\left(\frac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{1^2+(-2)^2+1^2}}\right)$

i.e., $=\left[(2\hat{i}-\hat{j}+\hat{k}).\left(\frac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{6}}\right)\right]\left(\frac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{6}}\right)=\left(\frac{2+2+1}{\sqrt{6}}\right)\left(\frac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{6}}\right)=\frac{5(\hat{i}-2\hat{j}+\hat{k})}{6}$.

Q12. As $P(B|A)=\frac{P(A\cap B)}{P(A)}$ $\Rightarrow P(A\cap B)=P(B|A)P(A)=0.6\times 0.4=0.24$

$\therefore P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{0.24}{0.8}=0.3$ or $\frac{3}{10}$.

SECTION C

Q13. Given that $\Delta=\begin{vmatrix}1 & a & a^2\\ a & a^2 & 1\\ a^2 & 1 & a\end{vmatrix}=-4$.

Consider C_{ij} be the cofactor of element a_{ij}.

Then $C_{11}=a^3-1,\ C_{12}=0,\ C_{13}=a-a^4;\ C_{21}=0,\ C_{22}=a-a^4,\ C_{23}=a^3-1;\ C_{31}=a-a^4,$ $C_{32}=1-a^3,\ C_{33}=0$.

So determinant formed by using the cofactors of Δ is $\begin{vmatrix}a^3-1 & 0 & a-a^4\\ 0 & a-a^4 & a^3-1\\ a-a^4 & a^3-1 & 0\end{vmatrix}=\Delta_1$ say.

As we know that $=\begin{vmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{vmatrix}=\Delta^{3-1}=\Delta^2$

(Here we've used $|adj.A|=|A|^{n-1}$, where n is order of A; also $|A|=|A^T|$.)

Hence $\Delta_1=(-4)^2=16$.

Alternatively, $\Delta=\begin{vmatrix}1 & a & a^2\\ a & a^2 & 1\\ a^2 & 1 & a\end{vmatrix}=-4$

By $C_1\to C_1-aC_3,\ C_3\to C_3-aC_2$,

$\Rightarrow \Delta=\begin{vmatrix}1-a^3 & a & 0\\ 0 & a^2 & 1-a^3\\ 0 & 1 & 0\end{vmatrix}=-4$

Expanding along C_1, we get : $\Delta=(1-a^3)[0-(1-a^3)]=-4 \Rightarrow (a^3-1)^2=4 \therefore (a^3-1)=\pm 2...(i)$.

Let $\Delta_1=\begin{vmatrix}a^3-1 & 0 & a-a^4\\ 0 & a-a^4 & a^3-1\\ a-a^4 & a^3-1 & 0\end{vmatrix}=(a^3-1)^3\begin{vmatrix}1 & 0 & -a\\ 0 & -a & 1\\ -a & 1 & 0\end{vmatrix}$

Taking a^3-1 common from R_1, R_2 and R_3 each.

$\Rightarrow \Delta_1=(a^3-1)^3\begin{vmatrix}1 & 0 & -a\\ 0 & -a & 1\\ -a & 1 & 0\end{vmatrix}$ Expanding along R_1

$\Rightarrow \Delta_1=(a^3-1)^3\left\{1(0-1)-0-a(0-a^2)\right\}=(a^3-1)^3\left\{a^3-1\right\}=(a^3-1)^4$

By (i), $\Delta_1=(\pm 2)^4=16$.

Q14. As f is differentiable at $x=1$ so, it is continuous at $x=1$ as well.

So, $\lim\limits_{x\to1^+} f(x)=\lim\limits_{x\to1^-} f(x)=f(1)$ i.e., $\lim\limits_{x\to1^-} ax^2+b=2(1)+1 \quad \Rightarrow a+b=3...(i)$

Also $Lf'(1)=Rf'(1)$ i.e., $\lim\limits_{x\to1^-}\dfrac{ax^2+b-3}{x-1}=\lim\limits_{x\to1^+}\dfrac{2x+1-3}{x-1}$

$\Rightarrow \lim\limits_{x\to1^-}\dfrac{ax^2-a}{x-1}=\lim\limits_{x\to1^+}\dfrac{2(x-1)}{x-1} \Rightarrow \lim\limits_{x\to1^-}\dfrac{a(x^2-1)}{x-1}=\lim\limits_{x\to1^+}2$ [By (i), $b-3=-a$

$\Rightarrow \lim\limits_{x\to1^-} a(x+1)=2 \quad \Rightarrow a(1+1)=2 \quad \therefore a=1,\ b=2$ [By (i).

OR It is given that f (x) is continuous at $x=0$.

So, $\lim\limits_{x\to0^+} f(x)=\lim\limits_{x\to0^-} f(x)=f(0)$ i.e., $\lim\limits_{x\to0^+} 2\dfrac{e^{\sin bx}-1}{bx}=\lim\limits_{x\to0^-}\dfrac{x+\sin x}{\sin(a+1)x}=2$

$\Rightarrow \lim\limits_{x\to0^+} 2\dfrac{e^{\sin bx}-1}{\sin bx}\times\dfrac{\sin bx}{bx}=\lim\limits_{x\to0^-}\dfrac{1+\dfrac{\sin x}{x}}{\dfrac{\sin(a+1)x}{(a+1)x}\times(a+1)}=2$ [As $x\to0\Rightarrow(a+1)x\to0$, $bx\to0,\ \sin bx\to0$

$\Rightarrow 2\times1\times1=\dfrac{1+1}{1\times(a+1)}=2$ i.e., $\dfrac{2}{a+1}=2 \quad \Rightarrow a=0$.

Hence $a=0$ and b may be any real number except 0 i.e., $b\in R-\{0\}$.

Q15. Here $y=\log\left(\sqrt{x}+\dfrac{1}{\sqrt{x}}\right)^2 \quad \Rightarrow y=2\log\left(\dfrac{x+1}{\sqrt{x}}\right)=2\log(x+1)-\log x$

$\therefore \dfrac{dy}{dx}=\dfrac{2}{x+1}-\dfrac{1}{x} \quad \Rightarrow \dfrac{dy}{dx}=\dfrac{x-1}{x(x+1)}\ldots(i)$

Again differentiating w.r.t. x both sides, we get : $\dfrac{d^2y}{dx^2}=\dfrac{x(x+1)(1-0)-(x-1)(2x+1)}{x^2(x+1)^2}$

$$x(x+1)^2\frac{d^2y}{dx^2}=\frac{x^2+x-2x^2+x+1}{x}=\frac{-x^2+2x+1}{x}$$

$$\Rightarrow x(x+1)^2\frac{d^2y}{dx^2}=\frac{2x-(x-1)(x+1)}{x}=2-\frac{(x-1)}{x(x+1)}\times(x+1)^2$$

By (i), we get : $x(x+1)^2\frac{d^2y}{dx^2}=2-\frac{dy}{dx}\times(x+1)^2$ $\quad\therefore x(x+1)^2y_2+(x+1)^2y_1=2$.

Alternatively, Here $y=\log\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$ $\quad\Rightarrow y=2\log\left(\frac{x+1}{\sqrt{x}}\right)=2\log(x+1)-\log x$

$\therefore\ \frac{dy}{dx}=\frac{2}{x+1}-\frac{1}{x}$ $\quad\Rightarrow\frac{dy}{dx}=\frac{x-1}{x(x+1)}$ $\quad\Rightarrow x\frac{dy}{dx}=\frac{x-1}{x+1}=1-\frac{2}{x+1}$

$\Rightarrow x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0+\frac{2}{(x+1)^2}$ $\quad\therefore x(x+1)^2\frac{d^2y}{dx^2}+(x+1)^2\frac{dy}{dx}=2$.

Q16. When the curve cuts x-axis, $y=0$ i.e., $(x^3-1)(x-2)=0$

$\Rightarrow(x-1)(x^2+x+1)(x-2)=0$ $\quad\therefore x=1,\ 2$ (as $x^2+x+1\neq0\ \forall\ x\in R$.

Hence points of contact are A(1, 0) and B(2, 0).

Now $\frac{dy}{dx}=(x^3-1)+3(x-2)x^2=4x^3-6x^2-1$ $\quad\therefore\left(\frac{dy}{dx}\right)_{at\ A}=-3,\ \left(\frac{dy}{dx}\right)_{at\ B}=7$.

Now equation of tangent at A : $y-0=-3(x-1)$ i.e., $3x+y=3$.

And equation of tangent at B : $y-0=7(x-2)$ i.e., $7x-y=14$.

OR Note that the domain of $f(x)=-3\log(1+x)+4\log(2+x)-\frac{4}{2+x}$ is $x\in(-1,\infty)$.

Now $f'(x)=-\frac{3}{1+x}+\frac{4}{2+x}+\frac{4}{(2+x)^2}=\frac{x(x+4)}{(1+x)(2+x)^2}$.

For $f'(x)=0,\ \frac{x(x+4)}{(1+x)(2+x)^2}=0\Rightarrow x=0,\ -4$. As $-4\in(-1,\infty)$ so, only $x=0$ is accepted.

As $f'(x)<0$ in $(-1,\ 0)$ so, f (x) is strictly decreasing in (–1, 0).

Also $f'(x)>0$ in $(0,\ \infty)$ so, f (x) is strictly increasing in $(0,\ \infty)$.

Q17. We've $C(x)=x^3-45x^2+600x$, $10\le x\le20$.

For the time being we may assume that the function C(x) is continuous at all the points in the interval [10, 20].

Now $C'(x)=3x^2-90x+600=3(x-10)(x-20)$, $C''(x)=6x-90$

For $C'(x)=3(x-10)(x-20)=0$ $\Rightarrow x=10,\ 20$.

Note that $C''(10)=-30<0$ and $C''(20)=30>0$.

So C(x) is minimum at x = 20 and maximum at x = 10.

Hence $C(10)=2500,\ C(20)=2000$.

Therefore the person must place the order for 20 trees in order to spend the least amount which is ₹ 2000.

Value Exhibited: Concern of the person for a healthy environment despite having economic constraints.

Q18. Let $I=\int\frac{\sec x}{1+\operatorname{cosec}x}dx=\int\frac{\sin x}{\cos x(1+\sin x)}dx=\int\frac{\sin x\cos x}{(1-\sin x)(1+\sin x)^2}dx$

Put $\sin x=t\Rightarrow\cos x dx=dt$ $\quad\therefore I=\int\frac{t}{(1-t)(1+t)^2}dt$

Consider $\frac{t}{(1-t)(1+t)^2}=\frac{A}{1+t}+\frac{B}{(1+t)^2}+\frac{C}{1-t}$ $\quad\Rightarrow t=A(1-t^2)+B(1-t)+C(1+t)^2$

On comparing the coefficients of like terms on both the sides, we get : $A=\frac{1}{4}, B=-\frac{1}{2}, C=\frac{1}{4}$.

So, $I=\int\left(\frac{1}{4}\times\frac{1}{1+t}-\frac{1}{2}\times\frac{1}{(1+t)^2}+\frac{1}{4}\times\frac{1}{1-t}\right)dt=\frac{1}{4}\log|1+t|+\frac{1}{2+2t}-\frac{1}{4}\log|1-t|+c$

$\therefore I=\frac{1}{4}\log\left|\frac{1+\sin x}{1-\sin x}\right|+\frac{1}{2+2\sin x}+c$.

Q19. We've $ye^y dx=(y^3+2xe^y)dy,\ y(0)=1$ $\Rightarrow \frac{dx}{dy}=\frac{y^3+2xe^y}{ye^y}$ $\Rightarrow \frac{dx}{dy}+\left(-\frac{2}{y}\right)x=\frac{y^2}{e^y}$

As this diff. eq. is in the form $\frac{dx}{dy}+P(y)x=Q(y)$ so, $P(y)=-\frac{2}{y},\ Q(y)=\frac{y^2}{e^y}$.

Now I.F. $=e^{\int-\frac{2}{y}dy}=e^{-2\log y}=\frac{1}{y^2}$.

Hence solution is, $x\left(\frac{1}{y^2}\right)=\int\frac{1}{y^2}\times\frac{y^2}{e^y}dy+C \Rightarrow \frac{x}{y^2}=-\frac{1}{e^y}+C$.

As $y(0)=1$ so, $\frac{0}{1^2}=-\frac{1}{e^1}+C \Rightarrow C=\frac{1}{e}$.

Hence the required particular solution is $\frac{x}{y^2}=-\frac{1}{e^y}+\frac{1}{e}$ or, $x=y^2(e^{-1}-e^{-y})$.

OR Here $(x-y)dy=(x+2y)dx$ $\Rightarrow \frac{dy}{dx}=\frac{x+2y}{x-y}=f(x,y)$ say …(i)

Let $x=\lambda x,\ y=\lambda y\ \therefore f(\lambda x,\lambda y)=\frac{\lambda x+2\lambda y}{\lambda x-\lambda y}=\lambda^0\left(\frac{x+2y}{x-y}\right)=\lambda^0 f(x,y)$.

Hence the differential equation is homogeneous.

Put $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ in (i)

Therefore, $v+x\frac{dv}{dx}=\frac{x+2vx}{x-vx}=\frac{1+2v}{1-v}$ $\Rightarrow x\frac{dv}{dx}=\frac{1+2v}{1-v}-v=\frac{1+2v-v+v^2}{1-v}$

$\Rightarrow x\frac{dv}{dx}=\frac{1+v+v^2}{1-v}$ $\Rightarrow \int\frac{1-v}{1+v+v^2}dv=\int\frac{dx}{x}$ $\Rightarrow -\frac{1}{2}\int\frac{-3+1+2v}{1+v+v^2}dv=\int\frac{dx}{x}$

$\Rightarrow \int\frac{-3}{1+v+v^2}dv+\int\frac{1+2v}{1+v+v^2}dv=-2\int\frac{dx}{x}$ $\Rightarrow \int\frac{-3}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dv+\int\frac{1+2v}{1+v+v^2}dv=-2\int\frac{dx}{x}$

$$\Rightarrow -3\times\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+\log|1+v+v^2|=-2\log|x|+C$$

$$\Rightarrow -2\sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right)+\log|1+v+v^2|=-2\log|x|+C$$

$$\Rightarrow \log|x^2+xy+y^2|-2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right)-2\log|x|=-2\log|x|+C$$

$$\therefore \log|x^2+xy+y^2|-2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right)=C.$$

Q20. Given $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ $\Rightarrow \vec{a}=-\vec{b}-\vec{c}$ $\Rightarrow \vec{a}\times\vec{b}=(-\vec{b}-\vec{c})\times\vec{b}=-\vec{b}\times\vec{b}-\vec{c}\times\vec{b}$

$\Rightarrow \vec{a}\times\vec{b}=-\vec{0}+\vec{b}\times\vec{c}$ $\Rightarrow \vec{a}\times\vec{b}=\vec{b}\times\vec{c}$ ...(i)

That is $\vec{a}\times\vec{b}=(-\vec{a}-\vec{c})\times\vec{c}$ $\left[\because\ \vec{a}+\vec{b}+\vec{c}=\vec{0}\ \Rightarrow \vec{b}=-\vec{a}-\vec{c}\right.$

$\Rightarrow \vec{a}\times\vec{b}=-\vec{a}\times\vec{c}-\vec{c}\times\vec{c}$ $\Rightarrow \vec{a}\times\vec{b}=\vec{c}\times\vec{a}-\vec{0}$ $\Rightarrow \vec{a}\times\vec{b}=\vec{c}\times\vec{a}$ ...(ii).

By (i) and (ii), $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

Now $[\vec{a}\ \vec{b}\ \vec{c}]=(\vec{a}\times\vec{b}).\vec{c}=(\vec{b}\times\vec{c}).\vec{c}=[\vec{b}\ \vec{c}\ \vec{c}]=0$. [By (i)

As the scalar triple product of three vectors is 0, if any two of them are equal vectors.

Q21. Let $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}=\lambda$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\mu$.

So, the coordinates of random points on these lines are $P(\lambda-2,\ 2\lambda+3,\ 4\lambda-1)$ and $Q(2\mu+1,\ 3\mu+2,\ 4\mu+3)$. Let A(1, 1, 1).

Now d.r.'s of line AP are $\lambda-3,\ 2\lambda+2,\ 4\lambda-2$ and, d.r.'s of line AQ are $2\mu,\ 3\mu+1,\ 4\mu+2$.

Therefore $\frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{4\lambda-2}{4\mu+2}=k$ say.

$\Rightarrow \lambda-3=2k\mu,\ 2\lambda+2=k(3\mu+1),\ 2\lambda-1=k(2\mu+1)$

$\Rightarrow \frac{\lambda-3}{2}=k\mu,\ 2\lambda+2=3\mu k+k,\ 2\lambda-1=2\mu k+k$

$\therefore 2\lambda+2=3\times\frac{\lambda-3}{2}+k,\ 2\lambda-1=2\times\frac{\lambda-3}{2}+k \Rightarrow \frac{\lambda+13}{2}=k,\ 2\lambda-1=\lambda-3+k$

$\therefore \lambda+2=\frac{\lambda+13}{2}$ $\Rightarrow 2\lambda+4=\lambda+13$ $\Rightarrow \lambda=9$

Also $\frac{9-3}{2\mu}=\frac{18+2}{3\mu+1}$ $\Rightarrow \frac{3}{2\mu}=\frac{10}{3\mu+1}$ $\Rightarrow \mu=\frac{3}{11}$

Therefore the d.r.'s of the required line through point A are 6, 20, 34 i.e., 3, 10, 17.

Hence the equation is : $\frac{x-1}{3}=\frac{y-1}{10}=\frac{z-1}{17}$.

Q22. Let E_1, E_2 and E_3 be the events that the bag I, bag II and bag III is chosen respectively.
Let E : the two balls drawn from the chosen bag are white and red.

Now $P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$, $P(E|E_1)=\frac{1}{6}\times\frac{3}{6}\times2,\ P(E|E_2)=\frac{2}{4}\times\frac{1}{4}\times2,\ P(E|E_3)=\frac{4}{9}\times\frac{2}{9}\times2$

By Bayes' theorem, $P(E_3|E)=\frac{P(E|E_3)P(E_3)}{\sum_{i=1}^{i=3}P(E|E_i)P(E_i)}=\frac{\frac{1}{3}\times\frac{4}{9}\times\frac{2}{9}\times2}{\frac{1}{3}\times\frac{1}{6}\times\frac{3}{6}\times2+\frac{1}{3}\times\frac{2}{4}\times\frac{1}{4}\times2+\frac{1}{3}\times\frac{4}{9}\times\frac{2}{9}\times2}$

$\therefore\ P(E_3|E)=\frac{64}{199}$.

Q23. Let X denotes the random variable. Then X can take values 0, 1, 2.
The probability distribution is as follow :

X	0	1	2
P(X)	$\frac{^{16}C_2}{^{20}C_2}=\frac{60}{95}$	$\frac{^{4}C_1\,^{16}C_1}{^{20}C_2}=\frac{32}{95}$	$\frac{^{4}C_2}{^{20}C_2}=\frac{3}{95}$

Now mean, $\mu=\sum X\,P(X)=0+\frac{32}{95}+\frac{6}{95}=\frac{38}{95}=\frac{2}{5}$.

Variance, $\sigma^2 = \sum X^2 P(X) - \mu^2 = 0 + \frac{32}{95} + \frac{12}{95} = \frac{44}{95} - \frac{4}{25} = \frac{144}{475}$.

SECTION D

Q24. $fog : \mathbb{R} \to \mathbb{R}$ defined by $fog(x) = f(g(x)) = f(x^3+5) = 2x^3+7 = h(x)$ say.

Now $h(x) = 2x^3 + 7$.

Let $x_1, x_2 \in \mathbb{R}$ (Domain of h(x)) such that $h(x_1) = h(x_2)$

i.e., $2x_1^3 + 7 = 2x_2^3 + 7 \Rightarrow x_1 = x_2$. Hence h (x) is one-one.

Let $y = h(x)$ such that $y \in \mathbb{R}$ (Codomain of h(x)).

That is $y = 2x^3 + 7 \quad \Rightarrow x = \left(\frac{y-7}{2}\right)^{1/3} \in \mathbb{R}$ (Domain of h (x)).

So for every $y \in \mathbb{R}$ there exists $x \in \mathbb{R}$ such that $h\left(\sqrt[3]{\frac{y-7}{2}}\right) = y$. Hence h (x) is onto.

As h (x) is one-one and onto so, it is invertible with $h^{-1}(x) = (fog)^{-1}(x) = \sqrt[3]{\frac{x-7}{2}}$.

Also $(fog)^{-1}(9) = \sqrt[3]{\frac{9-7}{2}} = 1$.

OR Let $a, b \in \mathbb{R}$ such that $a = 0, b \neq 0$.

Then $a*b = 0*b = |0| + b = 0 + b = b,\ b*a = b*0 = b \qquad \therefore a*b = b*a$.

Again let $a, b \in \mathbb{R}$ such that $a \neq 0, b = 0$.

Then $a*b = a*0 = a,\ b*a = 0*a = |0| + a = 0 + a = a \qquad \therefore a*b = b*a$.

For commutativity, let's check when $a \neq 0, b \neq 0$.

Then $a*b = |a| + b,\ b*a = |b| + a$. Clearly $a*b$ may not be equal to $b*a$.

For example, $(-1)*3 = |-1| + 3 = 4,\ 3*(-1) = |3| + (-1) = 2$, i.e., $(-1)*3 \neq 3*(-1)$,

$\therefore a*b \neq b*a \ \forall\ a, b \in \mathbb{R}$. Hence * is not commutative.

Let e be the identity element for *.

Then $a*e = e*a = a$ for all $a \in \mathbb{R}$.

Now $a*e = a$ if $e = 0$, and $e*a = a$ if $e = 0$.

(As $0*0 = 0$ and $0*a = |0| + a = a$ for $a \neq 0$)

Hence e = 0 is the identity element for *.

Q25. Here $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{vmatrix} = 3(3-6) - 2(-12-14) + 1(12+7) = 62 \neq 0$

Therefore A^{-1} exists.

Consider C_{ij} be the cofactor of a_{ij} for matrix A.

$C_{11} = -3, C_{12} = 26, C_{13} = 19;\ C_{21} = 9, C_{22} = -16, C_{23} = 5;\ C_{31} = 5, C_{32} = -2, C_{33} = -11$.

So, $adj.A = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix} \ldots(i)$

Now $3x + 4y + 7z = 14,\ 2x - y + 3z = 4,\ x + 2y - 3z = 0$

Let $P=\begin{pmatrix}3&4&7\\2&-1&3\\1&2&-3\end{pmatrix}=A^T$, $X=\begin{pmatrix}x\\y\\z\end{pmatrix}$, $B=\begin{pmatrix}14\\4\\0\end{pmatrix}$.

As $PX=B$ i.e., $X=(A^T)^{-1}B=(A^{-1})^TB$

So by (i), $X=\frac{1}{62}\begin{pmatrix}-3&26&19\\9&-16&5\\5&-2&-11\end{pmatrix}\begin{pmatrix}14\\4\\0\end{pmatrix}$

$\Rightarrow X=\frac{1}{62}\begin{pmatrix}62\\62\\62\end{pmatrix}$

$\Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\1\\1\end{pmatrix}$

$\therefore x=1, y=1, z=1$.

OR Here $A=\begin{bmatrix}2&1&1\\1&0&1\\0&2&-1\end{bmatrix}$

As $A=AI$ so, $\begin{bmatrix}2&1&1\\1&0&1\\0&2&-1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}A$ (By row transformations

By $R_1\leftrightarrow R_2$, $\begin{bmatrix}1&0&1\\2&1&1\\0&2&-1\end{bmatrix}=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}A$

By $R_2\to R_2-2R_1$, $\begin{bmatrix}1&0&1\\0&1&-1\\0&2&-1\end{bmatrix}=\begin{bmatrix}0&1&0\\1&-2&0\\0&0&1\end{bmatrix}A$

By $R_3\to R_3-2R_2$, $\begin{bmatrix}1&0&1\\0&1&-1\\0&0&1\end{bmatrix}=\begin{bmatrix}0&1&0\\1&-2&0\\-2&4&1\end{bmatrix}A$

By $R_2\to R_2+R_3$, $R_1\to R_1-R_3$, $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}2&-3&-1\\-1&2&1\\-2&4&1\end{bmatrix}A$

Since $I=A^{-1}A$

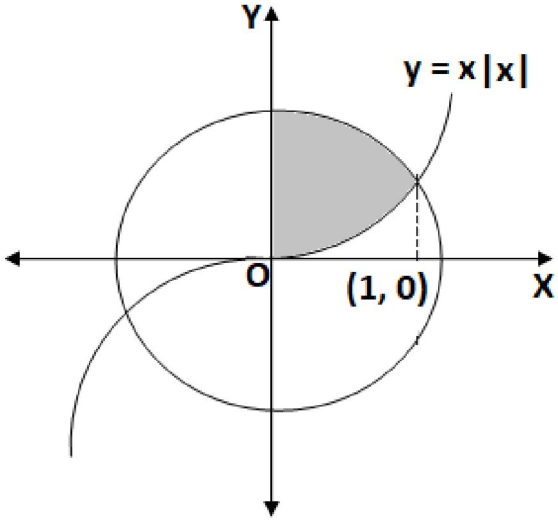
$\therefore A^{-1}=\begin{bmatrix}2&-3&-1\\-1&2&1\\-2&4&1\end{bmatrix}$.

Now $XA=\begin{bmatrix}1&0&1\end{bmatrix}$

$\Rightarrow XAA^{-1}=\begin{bmatrix}1&0&1\end{bmatrix}A^{-1}$

$\Rightarrow XI=\begin{bmatrix}1&0&1\end{bmatrix}A^{-1}$

$$\Rightarrow X = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$



Q26. We've $y = x|x| = \begin{cases} x^2, \text{ if } x \geq 0 \\ -x^2, \text{ if } x < 0 \end{cases}$.

And circle is $x^2 + y^2 = 2$.

Solving these simultaneously we get :

$y + y^2 = 2 \quad \Rightarrow (y-1)(y+2) = 0$

$\therefore y = 1$, as $y \neq -2$

So, $x^2 = 1 \therefore x = 1$ (in first quadrant)

$\therefore$ Point of intersection is (1, 1).

Required area $= \int_0^1 [\sqrt{2 - x^2} - x^2]dx$

$$\Rightarrow \quad = \left[\frac{x}{2}\sqrt{2 - x^2} + \frac{2}{2}\sin^{-1}\frac{x}{\sqrt{2}} \right]_0^1 - \frac{1}{3}\left(x^3\right)_0^1$$

$$\Rightarrow \quad = \left[\frac{1}{2} + \sin^{-1}\frac{1}{\sqrt{2}} \right] - 0 - \frac{1}{3}(1-0) = \frac{\pi}{4} + \frac{1}{6} \text{ sq.units}.$$

Q27. Let $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x}{2 - \cos 2x} dx + \frac{\pi}{4} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2 - \cos 2x} dx$.

Consider $f(x) = \frac{x}{2 - \cos 2x}$

$\Rightarrow f(-x) = \frac{-x}{2 - \cos 2(-x)} = -\frac{x}{2 - \cos 2x} = -f(x)$

And, $g(x) = \frac{1}{2 - \cos 2x}$

$\Rightarrow g(-x) = \frac{1}{2 - \cos 2(-x)} = \frac{1}{2 - \cos 2x} = g(x)$.

Clearly f (x) and g (x) are respectively odd and even functions.

By using $\int_{-a}^{a} f(x)dx = \begin{cases} 0, \text{ if } f(x) \text{ is odd} \\ 2\int_0^a f(x)dx, \text{ if } f(x) \text{ is even} \end{cases}$, we get :

$$I = 0 + \frac{\pi}{4} \times 2\int_0^{\frac{\pi}{4}} \frac{1}{2 - \cos 2x} dx$$

$$\Rightarrow I = \frac{\pi}{2}\int_0^{\frac{\pi}{4}} \frac{1}{2 - \frac{1 - \tan^2 x}{1 + \tan^2 x}} dx$$

$$\Rightarrow I = \frac{\pi}{2}\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{3\tan^2 x + 1} dx$$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$.

When $x = 0 \Rightarrow t = 0$, when $x = \frac{\pi}{4} \Rightarrow t = 1$.

So, $I = \frac{\pi}{2}\int_0^1 \frac{1}{3t^2+1}dt$

$\Rightarrow I = \frac{\pi}{2}\times\frac{1}{\sqrt{3}}\left[\tan^{-1}(\sqrt{3}\,t)\right]_0^1$

$\Rightarrow I = \frac{\pi}{2\sqrt{3}}\left[\tan^{-1}\sqrt{3} - 0\right] = \frac{\pi^2}{6\sqrt{3}}$.

OR Let $I = \int_{-2}^{2}(3x^2 - 2x + 4)dx$

We know $\int_a^b f(x)dx = \lim_{n\to\infty} h\left[f(a) + f(a+h) + f(a+2h) + ... + f(a+(n-1)h)\right]$,

As $n \to \infty,\ h \to 0 \Rightarrow nh = b - a$

$\therefore \int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh)$ ...(i)

Here $f(x) = 3x^2 - 2x + 4,\ a = -2,\ b = 2$.

$\therefore f(a+rh) = f(-2+rh) = 3(-2+rh)^2 - 2(-2+rh) + 4 = 3r^2h^2 - 14rh + 20$

By using (i), $\int_{-2}^{2}\left(3x^2 - 2x + 4\right)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1}\left[3r^2h^2 - 14rh + 20\right]$

$\Rightarrow I = \lim_{n\to\infty} h\left\{3h^2\sum_{r=0}^{n-1} r^2 - 14h\sum_{r=0}^{n-1} r + \sum_{r=0}^{n-1} 20\right\}$

$\Rightarrow I = \lim_{n\to\infty} h\left\{3h^2\sum_{r=0}^{n-1} r^2 - 14h\sum_{r=0}^{n-1} r + 20n\right\}$

$\Rightarrow I = \lim_{n\to\infty} h\left\{3h^2\left(\frac{n(n-1)(2n-1)}{6}\right) - 14h\left(\frac{n(n-1)}{2}\right) + 20n\right\}$

$\Rightarrow I = \lim_{n\to\infty}\left\{3\left(\frac{nh(nh-h)(2nh-h)}{6}\right) - 7nh(nh-h) + 20nh\right\}$

Since $n \to \infty,\ h \to 0 \Rightarrow nh = 2 - (-2) = 4$.

So, $I = 3\left(\frac{4(4-0)(8-0)}{6}\right) - 7\left(4(4-0)\right) + 20\times 4$

$\therefore I = 64 - 112 + 80 = 32$.

Hence $I = \int_{-2}^{2}(3x^2 - 2x + 4)dx = 32$.

Q28. The line $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 3\hat{j} - 9\hat{k})$ can be rewritten as $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+1}{-9} = \lambda$.

So coordinates of the random point on this line : $P(\lambda+1, 3\lambda+2, -9\lambda-1)$.

Let A(–2, 3, –4) be the point whose position vector is $-2\hat{i} + 3\hat{j} - 4\hat{k}$.

The d.r.'s of the line AP parallel to the plane $x - y + 2z - 3 = 0$ are $\lambda+3, 3\lambda-1, -9\lambda+3$.

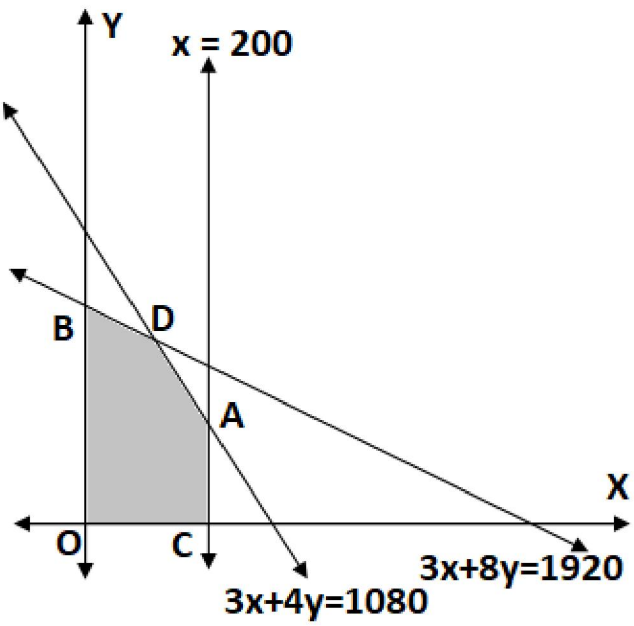
As normal to the plane shall be $\perp$ to line AP so, $1(\lambda+3) - (3\lambda-1) + 2(3-9\lambda) = 0$

$\Rightarrow \lambda = \frac{1}{2}$.

So $P\left(\frac{3}{2}, \frac{7}{2}, -\frac{11}{2}\right)$.

Therefore, the required distance $= \sqrt{\left(\frac{3}{2}+2\right)^2 + \left(\frac{7}{2}-3\right)^2 + \left(-\frac{11}{2}+4\right)^2} = \frac{\sqrt{59}}{2}$ units.

Q29. Let x and y be the no. of units of Product I and II to be produced daily, respectively.
To maximize : $Z = ₹\{(9-1.2)x + (8-0.9)y\} = ₹\,(7.8x + 7.1y)$.
Subject to the constraints : $x, y \geq 0$;
$\frac{x}{4} + \frac{y}{3} \leq 90$ i.e., $3x + 4y \leq 1080$,
$\frac{x}{8} + \frac{y}{3} \leq 80$ i.e., $3x + 8y \leq 1920$,
$x \leq 200$.



Corner Points	**Value Of Z (in ₹)**
O(0, 0)	0
A(200, 120)	2412 ← Max.Value
B(0, 240)	1704
C(200, 0)	1560
D(80, 210)	2115

So clearly, maximum profit is ₹2412.

This Sample Paper has been issued by **CBSE, New Delhi** for **2019 Board Exams of XII**.
Note : We've **re-typed** the same and have **added more illustrations** in the solutions.
On other hand, if you find any error which could have gone un-noticed, please do inform us via **WhatsApp @ +919650350480** or **Email us : iMathematicia@gmail.com**

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